<u>Negative / Rational Exponents</u>: When working on a problem, the **negative exponents need to be made positive.** To change negative exponents into positive exponents, write the reciprocal of the base and now write the exponent as positive. “**If the negative exponent is on top, move it down; if the negative exponent is on the bottom; move it up.:”** If the exponent is negative AND rational – <u>first make exponent positive</u>. Then evaluate rational exponent by writing a radical (**denominator = index; numerator = power**).

Examples:

x^{-2} ← Get the reciprocal, or move the negative exponent down.

$$x^{-2} = \frac{x^{-2}}{1} = \frac{1}{x^2}$$

$5y^{-3}$ ← Get the reciprocal of only the base with the negative exponent, the number stays in its place.

$$5y^{-3} = \frac{5y^{-3}}{1} = \frac{5}{y^3}$$

$\frac{a^2}{b^{-4}}$ ← Get the reciprocal of the base with the negative exponent, the base with the positive exponent stays in its place.

$$\frac{a^2}{b^{-4}} = \frac{a^2 \cdot b^4}{1} = a^2 \cdot b^4$$

$5^{-3} = \frac{1}{5^3}$ ←Write 5 under 1 and write exponent as positive. Evaluate.

$= \frac{1}{125}$

$$27^{\frac{-4}{3}}$$

$= \left(\frac{1}{27}\right)^{\frac{4}{3}}$ ← write reciprocal of base; write exponent as positive

$= \left(\sqrt[3]{\frac{1}{27}}\right)^4$ ←Change to radical. Denominator is index. Numerator is power.

$= \left(\frac{1}{3}\right)^4 = \frac{1}{81}$ ←Evaluate radical (cube root of 1 and of 27) then evaluate power (1^4 and 3^4)

Simplify by writing with positive exponent. Then simplify completely /find the value. Write final answer with positive exponents in variables.

1) 8^{-1} 2) 3^{-2} 5) $(3x)^{-1}$

7) $4c^{-3}$ 8) $2pr^{-5}$ 12) $\dfrac{5}{z^{-3}}$

13) $-\dfrac{2x}{a^{-4}}$ 14) $\dfrac{3b}{-5c^{-1}}$ 21) $\left(\dfrac{3}{4}\right)^{-1}$

22) $\left(\dfrac{2}{5}\right)^{-2}$ 23) $\left(\dfrac{2a}{9c}\right)^{-2}$ 24) $125^{-2/3}$ 25) $\left(\dfrac{9}{16}\right)^{-1/2}$

Answers:

1) $\dfrac{1}{8}$ 2) $\dfrac{1}{9}$ 5) $\dfrac{1}{3x}$ 7) $\dfrac{4}{c^3}$ 8) $\dfrac{2p}{r^5}$ 12) $5z^3$ 13) $-2xa^4$

14) $-\dfrac{3bc}{5}$ 21) $\dfrac{4}{3}$ 22) $\dfrac{25}{4}$ 23) $\dfrac{81c^2}{4a^2}$ 24) $\dfrac{1}{25}$ 25) $\dfrac{4}{3}$

Step	Method	Example
1	Label all unlabeled exponents "1"	$\left(\dfrac{25\ x^{-6}\ y\ (z^{-11})^2}{5\ (x^{-2})^5\ y^8\ z^2}\right)^{-2}$
2	**Get rid of any inside parentheses** by multiplying the exponents.	________________
3	**Combine all like bases** (*coefficients/ same letters*). -Divide coefficient/reduce fraction/leave as fraction. -Exponent law for variables: Top exponent minus bottom exponent, *if dividing.* (If bottom exponent is negative, remember –(-) = +.	________________
4	**Distribute outside exponents to all terms inside bracket.** Raise the coefficient to the exponent. Multiply the exponent by the variables' exponents.	________________
5	Move all negatives either up or down. Make the exponents positive.	________________
6	Find the value of the coefficient raised to an exponent.	________________

Now try this one : $\left(\dfrac{18x^{-4}\left(y^3\right)^{-2}(z^4)^2}{10(x^2)^{-3}y^4z^{-2}}\right)^{-2}$

A. To simplify these, use the index of the radical as the denominator of the exponent. Also find the root of the coefficient, if possible. Or write coefficient as mixed radical.

1. $\sqrt{25a^{18}b^{20}}$ 2. $\sqrt{8x^6y^8}$ 3. $\sqrt{x^{11}}$

4. $\sqrt[5]{\dfrac{x^5}{y^{10}}}$ 5. $\sqrt[3]{8x^4y^3}$ 6. $\sqrt[4]{81x^5y^2z^8}$

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B. True or False?

1. $9^{1/3} = \sqrt[3]{9}$
2. $8^{5/3} = \sqrt[5]{8^3}$
3. $(-16)^{1/2} = -16^{1/2}$
4. $9^{-3/2} = \dfrac{1}{27}$
5. $6^{-1/2} = \; = \dfrac{1}{\sqrt{6}}$
6. $\dfrac{2}{2^{1/2}} = 2^{1/2}$
7. $2^{1/2} \cdot 2^{1/2} = 4^{1/2}$
8. $16^{-1/4} = -2$
9. $6^{1/6} \cdot 6^{1/6} = 6^{1/3}$
10. $(2^8)^{3/4} = 2^6$

Answers

A 1). $5a^9b^{10}$ 2). $2\sqrt{2}\,x^3y^4$ 3) $x^{\frac{11}{2}}$ 4) $\frac{x}{y^2}$ 5) $2x^{\frac{4}{3}}$y 6) $3x^{\frac{5}{4}}y^{\frac{1}{2}}z^2$

B. 1. T 2. F 3. F 4. T 5. T 6. T 7. T 8. F 9. T 10. T